

M.Sc. (Mathematics) (NEP Pattern) Semester-II
Major Elective DSE-3 - Combinatorics

P. Pages : 2

Time : Three Hours



GUG/S/25/15398

Max. Marks : 80

- Notes : 1. Solve **all five** questions.
2. Each question carry equal marks.

UNIT – I

1. a) How many arrangements of the seven letters in the word SYSTEMS have the E occurring somewhere before the M? How many arrangements have the E somewhere before the M and the three Ss grouped consecutively? **8**
- b) There are five different Spanish books, six different French books, and eight different Transylvanian books. How many ways are there to pick an unordered pair of two books not both in the same language? **8**

OR

- c) A committee of k people is to be chosen from a set of seven women and four men. How many ways are there to form the committee if **8**
- (a) The committee consists of three women and two men?
- (b) The committee can be any positive size but must have equal numbers of women and men?
- (c) The committee has four people and one of them must be Mr. Baggins?
- (d) The committee has four people and at least two are women?
- d) What is the probability that a 4-digit campus telephone number has one or more repeated digits? **8**

UNIT – II

2. a) Find the coefficient of x^{16} in **8**
- $$\left(x + x^2 + x^3 + x^4 + x^5\right)\left(x^2 + x^3 + x^4 + \dots\right)^5$$
- b) Use generating functions to find the number of ways to collect \$15 from 20 distinct people if each of the first 19 people can give a dollar (or nothing) and the twentieth person can give either \$1 or \$5 (or nothing). **8**

OR

- c) How many ways are there to distribute 25 identical balls into seven distinct boxes if the first box can have no more than 10 balls but any number can go into each of the other six boxes? **8**
- d) How many ways are there to select 25 toys from seven types of toys with between one and six of each type? **8**

UNIT – III

3. a) Solve the recurrence relation $a_n = a_{n-2}$ with $a_0 = a_1 = 1$. 8
- b) Solve the recurrence relation
 $a_n = 3a_{n-1} - 3a_{n-2} + a_{n-3}$, $a_0 = a_1 = 1$, $a_2 = 2$ 8
- OR**
- c) Solve the recurrence relation $a_n = a_{n-1} + 3n^2$ with $a_0 = 10$. 8
- d) Solve the recurrence relation $a_n = a_{n-1} + a_{n-2}$ with $a_0 = a_1 = 1$. 8

UNIT – IV

4. a) How many ways are there to distribute r distinct objects into five distinct boxes with at least one empty box? 8
- b) let A_1, A_2, \dots, A_n , be n sets in a universe u of N elements. Let S_k denote the sum of the sizes of all k -tuple intersections of the A_i s.
Then
$$N(\overline{A_1} \cap \overline{A_2} \cap \dots \cap \overline{A_n}) = N - S_1 + S_2 - S_3 + \dots + (-1)^k S_k + \dots + (-1)^n S_n$$
- OR**
- c) If a school has 80 students with 40 taking French, 40 taking Latin, and 40 taking German, 20 students are taking any given pair of languages, and 10 students are taking all three languages, then how many students are taking no language? 8
- d) How many ways are there to select a 6-card hand from a regular 52-card deck such that the hand contains at least one card in each suit? 8
How many 6-card hands with a void in at least one suit?
5. Solve all the four questions.
- a) State Addition Principle and Multiplication Principle. 4
- b) Find the generating function for a_r , the number of ways to select r objects chosen from three types of objects with repetition of up to four objects of each type. 4
- c) A bank pays 4 percent interest each year on money in savings accounts. Find recurrence relations for the amounts of money a person would have after n years if he follows the investment strategies of 4
- (a) Investing 1000 Rs. and leaving it in the bank for n years.
- (b) Investing 100 Rs. at the end of each year.
- d) How many positive integers ≤ 100 are relatively prime to 100? 4
